SUMMARY: FUNDAMENTALS OF COLLEGE PHYSICS

Conversion of units

THE ROLE OF UNITS IN PROBLEM SOLVING To convert a number from one unit to another, multiply the number by the ratio of the two units.

Dimension

The dimension of a quantity represents its physical nature and the type of unit used to specify it. Three such dimensions are length [L], mass [M], time [T].

Sine, cosine, and tangent of an angle θ

TRIGONOMETRY The sine, cosine, and tangent functions of an angle θ are defined in terms of a right triangle that contains θ:

$$\sin \theta = \frac{h_o}{h} \qquad \cos \theta = \frac{h_a}{h} \qquad \tan \theta = \frac{h_o}{h_a}$$

where h_o and h_a are, respectively, the lengths of the sides opposite and adjacent to the angle θ, and h is the length of the hypotenuse.

Inverse trigonometric functions

The inverse sine, inverse cosine, and inverse tangent functions are

$$\theta = \sin^{-1}\left(\frac{h_o}{h}\right) \qquad \theta = \cos^{-1}\left(\frac{h_a}{h}\right) \qquad \theta = \tan^{-1}\left(\frac{h_o}{h_a}\right)$$

Pythagorean theorem

The Pythagorean theorem states that the square of the length of the hypotenuse of a right triangle is equal to the sum of the squares of the lengths of the other two sides:

$$h^2 = h_o{}^2 + h_a{}^2$$

Scalars and vectors

SCALARS AND VECTORS A scalar quantity is described completely by its size, which is also called its magnitude. A vector quantity has both a magnitude and a direction. Vectors are often represented by arrows, the length of the arrow being proportional to the magnitude of the vector and the direction of the arrow indicating the direction of the vector.

Graphical method of vector addition and subtraction

VECTOR ADDITION AND SUBTRACTION One procedure for adding vectors utilizes a graphical technique, in which the vectors to be added are arranged in a tail-to-head fashion. The resultant vector is drawn from the tail of the first vector to the head of the last vector.

The subtraction of a vector is treated as the addition of a vector that has been multiplied by a scalar factor of -1. Multiplying a vector by -1 reverses the direction of the vector.

Vector components

THE COMPONENTS OF A VECTOR In two dimensions, the vector components of a vector $\vec{A}$ are two perpendicular vectors $\vec{A}_x$ and $\vec{A}_y$ that are parallel to the x and y axes, respectively, and that add together vectorially so that $\vec{A} = \vec{A}_x + \vec{A}_y$.

Scalar components

The scalar component A_x has a magnitude that is equal to that of $\vec{A}_x$ and is given a positive sign if $\vec{A}_x$ points along the $+x$ axis and a negative sign if $\vec{A}_x$ points along the $-x$ axis. The scalar component A_y is defined in a similar manner.

Condition for a vector to be zero

A vector is zero if, and only if, each of its vector components is zero.

Condition for two vectors to be equal

Two vectors are equal if, and only if, they have the same magnitude and direction. Alternatively, two vectors are equal in two dimensions if the x vector components of each are equal and the y vector components of each are equal.

ADDITION OF VECTORS BY MEANS OF COMPONENTS If two vectors $\vec{A}$ and $\vec{B}$ are added to give a resultant $\vec{C}$ such that $\vec{C} = \vec{A} + \vec{B}$, then

$$C_x = A_x + B_x \quad \text{and} \quad C_y = A_y + B_y$$

where C_x, A_x, and B_x are the scalar components of the vectors along the x direction, and C_y, A_y, and B_y are the scalar components of the vectors along the y direction.

Kinematics

Topic	Discussion

Displacement vector

DISPLACEMENT, VELOCITY, AND ACCELERATION The position of an object is located with a vector $\vec{r}$ drawn from the coordinate origin to the object. The displacement $\Delta\vec{r}$ of the object is defined as $\Delta\vec{r} = \vec{r} - \vec{r}_0$, where $\vec{r}$ and $\vec{r}_0$ specify its final and initial positions, respectively.

Average velocity

The average velocity $\overline{\vec{v}}$ of an object moving between two positions is defined as its displacement $\Delta\vec{r} = \vec{r} - \vec{r}_0$ divided by the elapsed time $\Delta t = t - t_0$:

$$\overline{\vec{v}} = \frac{\vec{r} - \vec{r}_0}{t - t_0} = \frac{\Delta\vec{r}}{\Delta t}$$

Instantaneous velocity

The instantaneous velocity $\vec{v}$ is the velocity at an instant of time. The average velocity becomes equal to the instantaneous velocity in the limit that Δt becomes infinitesimally small ($\Delta t \to 0$ s):

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta\vec{r}}{\Delta t}$$

ACCELERATION The average acceleration $\overline{\vec{a}}$ is a vector. It equals the change $\Delta\vec{v}$ in the velocity divided by the elapsed time Δt, the change in the velocity being the final minus the initial velocity:

Average acceleration

$$\overline{\vec{a}} = \frac{\Delta\vec{v}}{\Delta t}$$

When Δt becomes infinitesimally small, the average acceleration becomes equal to the instantaneous acceleration $\vec{a}$:

Instantaneous acceleration

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t}$$

Acceleration is the rate at which the velocity is changing.

Acceleration due to gravity

FREELY FALLING BODIES In free-fall motion, an object experiences negligible air resistance and a constant acceleration due to gravity. All objects at the same location above the earth have the same acceleration due to gravity. The acceleration due to gravity is directed toward the center of the earth and has a magnitude of approximately 9.80 m/s^2 near the earth's surface.

EQUATIONS OF KINEMATICS IN TWO DIMENSIONS Motion in two dimensions can be described in terms of the time t and the x and y components of four vectors: the displacement, the acceleration, and the initial and final velocities.

Independence of the *x* and *y* parts of the motion

The x part of the motion occurs exactly as it would if the y part did not occur at all. Similarly, the y part of the motion occurs exactly as it would if the x part of the motion did not exist. The motion can be analyzed by treating the x and y components of the four vectors separately and realizing that the time t is the same for each component.

Equations of kinematics for constant acceleration

When the acceleration is constant, the x components of the displacement, the acceleration, and the initial and final velocities are related by the equations of kinematics, and so are the y components:

x Component	y Component
$v_x = v_{0x} + a_x t$	$v_y = v_{0y} + a_y t$
$x = \frac{1}{2}(v_{0x} + v_x)t$	$y = \frac{1}{2}(v_{0y} + v_y)t$
$x = v_{0x}t + \frac{1}{2}a_x t^2$	$y = v_{0y}t + \frac{1}{2}a_y t^2$
$v_x^2 = v_{0x}^2 + 2a_x x$	$v_y^2 = v_{0y}^2 + 2a_y y$

The directions of these components are conveyed by assigning a plus ($+$) or minus ($-$) sign to each one.

Dynamics, Forces and Newton's Laws of Motion

Topic	Discussion
Contact and noncontact forces	**THE CONCEPTS OF FORCE AND MASS** A force is a push or a pull and is a vector quantity. Contact forces arise from the physical contact between two objects. Noncontact forces are also called action-at-a-distance forces, because they arise without physical contact between two objects.
Mass	Mass is a property of matter that determines how difficult it is to accelerate or decelerate an object. Mass is a scalar quantity.
Newton's first law	**NEWTON'S FIRST LAW OF MOTION** Newton's first law of motion, sometimes called the law of inertia, states that an object continues in a state of rest or in a state of motion at a constant velocity unless compelled to change that state by a net force.
Inertia Mass Inertial reference frame	Inertia is the natural tendency of an object to remain at rest or in motion at a constant velocity. The mass of a body is a quantitative measure of inertia and is measured in an SI unit called the kilogram (kg). An inertial reference frame is one in which Newton's law of inertia is valid.

NEWTON'S SECOND LAW OF MOTION   **THE VECTOR NATURE OF NEWTON'S SECOND LAW OF MOTION** Newton's second law of motion states that when a net force $\Sigma\vec{F}$ acts on an object of mass m, the acceleration $\vec{a}$ of the object can be obtained from the following equation:

Newton's second law (vector form)

$$\Sigma\vec{F} = m\vec{a}$$

This is a vector equation and, for motion in two dimensions, is equivalent to the following two equations:

Newton's second law (component form)

$$\Sigma F_x = ma_x$$
$$\Sigma F_y = ma_y$$

In these equations the x and y subscripts refer to the scalar components of the force and acceleration vectors. The SI unit of force is the Newton (N).

Free-body diagram	When determining the net force, a free-body diagram is helpful. A free-body diagram is a diagram that represents the object and the forces acting on it.
Newton's third law of motion	**NEWTON'S THIRD LAW OF MOTION** Newton's third law of motion, often called the action–reaction law, states that whenever one object exerts a force on a second object, the second object exerts an oppositely directed force of equal magnitude on the first object.

THE GRAVITATIONAL FORCE Newton's law of universal gravitation states that every particle in the universe exerts an attractive force on every other particle. For two particles that are separated by a distance r and have masses m_1 and m_2, the law states that the magnitude of this attractive force is

Newton's law of universal gravitation

$$F = G\frac{m_1 m_2}{r^2}$$

The direction of this force lies along the line between the particles. The constant G has a value of $G = 6.674 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2$ and is called the universal gravitational constant.

The weight W of an object on or above the earth is the gravitational force that the earth exerts on the object and can be calculated from the mass m of the object and the magnitude g of the acceleration due to the earth's gravity according to

Weight and mass

$$W = mg$$

Normal force

THE NORMAL FORCE The normal force $\vec{F}_N$ is one component of the force that a surface exerts on an object with which it is in contact—namely, the component that is perpendicular to the surface.

STATIC AND KINETIC FRICTIONAL FORCES A surface exerts a force on an object with which it is in contact. The component of the force perpendicular to the surface is called the normal force. The component parallel to the surface is called friction.

Friction

The force of static friction between two surfaces opposes any impending relative motion of the surfaces. The magnitude of the static frictional force depends on the magnitude of the applied force and can assume any value up to a maximum of

Maximum static frictional force

$$f_s^{\text{MAX}} = \mu_s F_N$$

where μ_s is the coefficient of static friction and F_N is the magnitude of the normal force.

The force of kinetic friction between two surfaces sliding against one another opposes the relative motion of the surfaces. This force has a magnitude given by

Kinetic frictional force

$$f_k = \mu_k F_N$$

where μ_k is the coefficient of kinetic friction.

Tension

THE TENSION FORCE The word "tension" is commonly used to mean the tendency of a rope to be pulled apart due to forces that are applied at each end. Because of tension, a rope transmits a force from one end to the other. When a rope is accelerating, the force is transmitted undiminished only if the rope is massless.

Definition of equilibrium

EQUILIBRIUM APPLICATIONS OF NEWTON'S LAWS OF MOTION An object is in equilibrium when the object has zero acceleration, or, in other words, when it moves at a constant velocity (which may be zero). The sum of the forces that act on an object in equilibrium is zero. Under equilibrium conditions in two dimensions, the separate sums of the force components in the x direction and in the y direction must each be zero:

The equilibrium condition

$$\Sigma F_x = 0$$
$$\Sigma F_y = 0$$

NONEQUILIBRIUM APPLICATIONS OF NEWTON'S LAWS OF MOTION If an object is not in equilibrium, then Newton's second law must be used to account for the acceleration:

$$\Sigma F_x = ma_x$$
$$\Sigma F_y = ma_y$$

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Physical Quantity	Letter of Physical Variable	Name of Unit	Unit Symbol /Abbreviation/
temperature	T	kelvin	**K**
		degree Celsius	°C
mass	m	kilogram	**kg**
time	t	second	**s**
length, distance	l or s or d	meter	**m**
displacement	s	meter	m
height, depth	h	meter	m
speed, velocity	v	meter per second	m/s (or m·s^{-1})
change of velocity	Δv	meter per second	m/s
average velocity	$\bar{v}$	meter per second	m/s
initial velocity	v_0	meter per second	m/s
acceleration	a	meter per second per second	m/s^2 (or m·s^{-2})
area	A	square meter	m^2
volume	V	cubic meter	m^3
density	ρ	kilogram per cubic meter	kg/m^3 or (kg·m^{-3})
force, tension, upthrust	F	newton	N (= kg·m/s^2)
weight	F_w	newton	N (= kg·m/s^2)
acceleration due to gravity	g	meter per second per second	m/s^2
impulse, momentum	p	kilogram meter per second	kg·m/s
energy	E	joule	J (= N·m)
work done	W	joule	J (= N·m)
potential energy	E_p or PE	joule	J (= N·m)
kinetic energy	E_k or KE	joule	J (= N·m)
power	P	watt	W (= J/s)
pressure	P or p	pascal	Pa (= N/m^2)

Dynamics of Uniform Circular Motion

Topic	Discussion
	UNIFORM CIRCULAR MOTION Uniform circular motion is the motion of an object traveling at a constant (uniform) speed on a circular path.
Period of the motion	The period T is the time required for the object to travel once around the circle. The speed v of the object is related to the period and the radius r of the circle by $$v = \frac{2\pi r}{T}$$
	CENTRIPETAL ACCELERATION An object in uniform circular motion experiences an acceleration, known as centripetal acceleration. The magnitude a_c of the centripetal acceleration is
Centripetal acceleration (magnitude)	$$a_c = \frac{v^2}{r}$$
Centripetal acceleration (direction)	where v is the speed of the object and r is the radius of the circle. The direction of the centripetal acceleration vector always points toward the center of the circle and continually changes as the object moves.
	CENTRIPETAL FORCE To produce a centripetal acceleration, a net force pointing toward the center of the circle is required. This net force is called the centripetal force, and its magnitude F_c is
Centripetal force (magnitude)	$$F_c = \frac{mv^2}{r}$$
Centripetal force (direction)	where m and v are the mass and speed of the object, and r is the radius of the circle. The direction of the centripetal force vector, like that of the centripetal acceleration vector, always points toward the center of the circle.

Work and Energy

Topic	Discussion
	WORK DONE BY A CONSTANT FORCE The work W done by a constant force acting on an object is
Work done by constant force	$$W = (F \cos \theta)s$$
	where F is the magnitude of the force, s is the magnitude of the displacement, and θ is the angle between the force and the displacement vectors. Work is a scalar quantity and can be positive or negative, depending on whether the force has a component that points, respectively, in the same direction as the displacement or in the opposite direction. The work is zero if the force is perpendicular ($\theta = 90°$) to the displacement.
	THE WORK–ENERGY THEOREM AND KINETIC ENERGY The kinetic energy KE of an object of mass m and speed v is
Kinetic energy	$$KE = \tfrac{1}{2}mv^2$$
	The work–energy theorem states that the work W done by the net external force acting on an object equals the difference between the object's final kinetic energy KE_f and initial kinetic energy KE_0:
Work-energy theorem	$$W = KE_f - KE_0$$
	The kinetic energy increases when the net force does positive work and decreases when the net force does negative work.
	GRAVITATIONAL POTENTIAL ENERGY The work done by the force of gravity on an object of mass m is
Work done by force of gravity	$$W_{gravity} = mg(h_0 - h_f)$$
	where h_0 and h_f are the initial and final heights of the object, respectively.
	Gravitational potential energy PE is the energy that an object has by virtue of its position. For an object near the surface of the earth, the gravitational potential energy is given by
Gravitational potential energy	$$PE = mgh$$
	where h is the height of the object relative to an arbitrary zero level.
Conservative force	**CONSERVATIVE VERSUS NONCONSERVATIVE FORCES** A conservative force is one that does the same work in moving an object between two points, independent of the path taken between the points. Alternatively, a force is conservative if the work it does in moving an object around any closed path is zero. A force is nonconservative if the work it does on an object moving between
Nonconservative force	two points depends on the path of the motion between the points.
	THE CONSERVATION OF MECHANICAL ENERGY The total mechanical energy E is the sum of the kinetic energy and potential energy:
Total mechanical energy	$$E = KE + PE$$
	The work–energy theorem can be expressed in an alternate form as
Alternate form for work-energy theorem	$$W_{nc} = E_f - E_0$$
	where W_{nc} is the net work done by the external nonconservative forces, and E_f and E_0 are the final and initial total mechanical energies, respectively.
Principle of conservation of mechanical energy	The principle of conservation of mechanical energy states that the total mechanical energy E remains constant along the path of an object, provided that the net work done by external nonconservative forces is zero. Whereas E is constant, KE and PE may be transformed into one another.

Rotatinal Kinematics

Topic	Discussion
Angular displacement	**ROTATIONAL MOTION AND ANGULAR DISPLACEMENT** When a rigid body rotates about a fixed axis, the angular displacement is the angle swept out by a line passing through any point on the body and intersecting the axis of rotation perpendicularly. By convention, the angular displacement is positive if it is counterclockwise and negative if it is clockwise.
Radian	The radian (rad) is the SI unit of angular displacement. In radians, the angle θ is defined as the circular arc of length s traveled by a point on the rotating body divided by the radial distance r of the point from the axis: $$\theta \text{ (in radians)} = \frac{s}{r}$$
Average angular velocity	**ANGULAR VELOCITY AND ANGULAR ACCELERATION** The average angular velocity $\overline{\omega}$ is the angular displacement $\Delta\theta$ divided by the elapsed time Δt: $$\overline{\omega} = \frac{\Delta\theta}{\Delta t}$$
Instantaneous angular velocity	As Δt approaches zero, the average angular velocity becomes equal to the instantaneous angular velocity ω. The magnitude of the instantaneous angular velocity is called the instantaneous angular speed.
Average angular acceleration	The average angular acceleration $\overline{\alpha}$ is the change $\Delta\omega$ in the angular velocity divided by the elapsed time Δt: $$\overline{\alpha} = \frac{\Delta\omega}{\Delta t}$$
Instantaneous angular acceleration	As Δt approaches zero, the average angular acceleration becomes equal to the instantaneous angular acceleration α.
Equations of rotational kinematics	**THE EQUATIONS OF ROTATIONAL KINEMATICS** The equations of rotational kinematics apply when a rigid body rotates with a constant angular acceleration about a fixed axis. These equations relate the angular displacement $\theta - \theta_0$, the angular acceleration α, the final angular velocity ω, the initial angular velocity ω_0, and the elapsed time $t - t_0$. Assuming that $\theta_0 = 0$ rad at $t_0 = 0$ s, the equations of rotational kinematics are $$\omega = \omega_0 + \alpha t$$ $$\theta = \tfrac{1}{2}(\omega + \omega_0)t$$ $$\theta = \omega_0 t + \tfrac{1}{2}\alpha t^2$$ $$\omega^2 = \omega_0{}^2 + 2\alpha\theta$$ These equations may be used with any self-consistent set of units and are not restricted to radian measure.
Relations between angular and tangential variables	**ANGULAR VARIABLES AND TANGENTIAL VARIABLES** When a rigid body rotates through an angle θ about a fixed axis, any point on the body moves on a circular arc of length s and radius r. Such a point has a tangential velocity (magnitude $= v_\mathrm{T}$) and, possibly, a tangential acceleration (magnitude $= a_\mathrm{T}$). The angular and tangential variables are related by the following equations: $$s = r\theta \qquad (\theta \text{ in rad})$$ $$v_\mathrm{T} = r\omega \qquad (\omega \text{ in rad/s})$$ $$a_\mathrm{T} = r\alpha \qquad (\alpha \text{ in rad/s}^2)$$ These equations refer to the magnitudes of the variables involved, without reference to positive or negative signs, and only radian measure can be used when applying them.

Rotational Dynamics

Topic	Discussion
Line of action Lever arm	**THE ACTION OF FORCES AND TORQUES ON RIGID OBJECTS** The line of action of a force is an extended line that is drawn colinear with the force. The lever arm ℓ is the distance between the line of action and the axis of rotation, measured on a line that is perpendicular to both.
Torque	The torque of a force has a magnitude that is given by the magnitude F of the force times the lever arm ℓ. The torque τ is $$\tau = F\ell$$ and is positive when the force tends to produce a counterclockwise rotation about the axis, and negative when the force tends to produce a clockwise rotation.
Equilibrium of a rigid body	**RIGID OBJECTS IN EQUILIBRIUM** A rigid body is in equilibrium if it has zero translational acceleration and zero angular acceleration. In equilibrium, the net external force and the net external torque acting on the body are zero: $$\Sigma F_x = 0 \quad \text{and} \quad \Sigma F_y = 0$$ $$\Sigma \tau = 0$$
Definition of center of gravity	**CENTER OF GRAVITY** The center of gravity of a rigid object is the point where its entire weight can be considered to act when calculating the torque due to the weight. For a symmetrical body with uniformly distributed weight, the center of gravity is at the geometrical center of the body. When a number of objects whose weights are $W_1, W_2, \ldots$ are distributed along the x axis at locations $x_1, x_2, \ldots$, the center of gravity x_{cg} is located at $$x_{cg} = \frac{W_1 x_1 + W_2 x_2 + \cdots}{W_1 + W_2 + \cdots}$$ The center of gravity is identical to the center of mass, provided the acceleration due to gravity does not vary over the physical extent of the objects.
Moment of inertia	**NEWTON'S SECOND LAW FOR ROTATIONAL MOTION ABOUT A FIXED AXIS** The moment of inertia I of a body composed of N particles is $$I = m_1 r_1^2 + m_2 r_2^2 + \cdots + m_N r_N^2 = \Sigma mr^2$$ where m is the mass of a particle and r is the perpendicular distance of the particle from the axis of rotation.
Newton's second law for rotational motion	For a rigid body rotating about a fixed axis, Newton's second law for rotational motion is $$\Sigma \tau = I\alpha \quad (\alpha \text{ in rad/s}^2)$$ where $\Sigma \tau$ is the net external torque applied to the body, I is the moment of inertia of the body, and α is its angular acceleration.